The Teaching of Mathematics

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WHY do we teach mathematics? This question was recently posed in the columns of the Mathematical Gazette, which is the organ of the methematics teachers in Great Britain. It is indeed a question which must be uppermost in the minds of all teachers, for what we teach and how we teach it can only be decided in the light of the reply to this question.

In the article referred to, the point was made that mathematics is becoming an essential tool for more and more people. This is indeed so. In many schools the pupils interested in science, but lacking in mathematical ability, are encouraged to study biology, because this subject does not require mathematics. This policy is a dangerous one, because mathematics is now almost as indispensable to the research worker in the various biological fields as it is to the chemist and the physicist. The same can be said about economics and the social sciences. If we take a larger view however, even this widening of the application of mathematics can only affect a comparatively small proportion of the pupils who pass through our elementary and secondary schools. Even today it is doubtful whether a very strong case can be made out, on grounds of immediate practical utility alone, for the teaching of much more than the basic arithmetic of integers, fractions, decimals and percentages and of the mensuration of the simplest geometrical figures in any but the examination streams of the Grammar Schools. Nor indeed would many people attempt to do so. When it comes to deciding the content of the mathematics syllabus the ever widening application of mathematics must however not be overlooked. The syllabus will have to be planned with the more modern applications of mathematics in view.

It becomes ever more difficult also to give an account of the current developments in science without using at least some mathematical ideas. The person with only a rudimentary mathematical education is therefore in danger of being more and more cut off from the main advances of our civilisation. He may be condemned to live in a world, which he is not equipped to understand, a world in which much seems like magic, and in which he has often to be guided by uncomprehended authority. Work which is designed merely to cover one of the syllabuses for the ordinary level mathematics examination of the General Certificate of Education comes, in the writer's opinion, under the heading of 'rudimentary mathematical education'. For a person who has only reached this level, science would have to assume the role which the Church played for the common man in the middle ages.

Although these considerations have already a wider application than those of utility, it may be doubted whether many teachers would regard them as the main reasons for teaching mathematics. When asked by a pupil why he has to do sums, the teacher would probably reply, "Because it is good for you", or, "Because it is a good training for your mind". Before we can discuss a mathematics curriculum or the way in which the subject should be presented at the various ages, we must have a clear idea whether the teacher's reply is in fact true, and if so, why, and under what conditions it is true. To do this it will be necessary to look more closely at mathematics itself and at the mental and psychological development of the child.

An examination of the application of mathematics and of its relationship to science can teach us much about the nature of the subject. Shortly before his death the celebrated physicist Sir Arthur Eddington published a book about the philosophical basis of physics.* He comes to the conclusion that physics can only discover the structure of the physical universe, in so far as this can be embodied in mathematical equations. What the equations describe, the contents of the structure, he maintains has not been and cannot be discovered by the methods which physics uses at present. It is as if on investigaing a building we could only discover its dimensions and the size and arrangement of the fundamental units, but not the material of which it is made.

At first sight such a statement may seem to be in violent contradiction to the actual facts. In recent years we have often been made aware only too acutely that the applications of our physical knowledge have nothing of a merely theoretical nature about them. We need only consider the rapidity with which the face of the civilised world has changed in the last decades, to obtain tangible proof of the power of the methods which Eddington believed had such severe limitations. And yet a posthumously published work by the same author,** which is highly mathematical, goes far to substantiate the claims in the author's earlier book. In any case the opinions of so eminent an exponent of modern physics deserve to be taken seriously.

Before drawing any conclusions let us turn to another realm in which mathematics has advanced by leaps and bounds in recent years. I am referring to statistics. In some more popular treatises the chapter dealing with probability and statistics is entitled 'Chance'. What could on first sight be further apart than strict, logical mathematics and fortuitous chance, in which by definition there is no law, no rhyme nor reason? And yet the application of mathematics to apparently quite unordered events has been eminently successful. The balance sheets of insurance companies and the finances of Monte Carlo bear witness to this fact. Closer acquaintance with probability calculus makes the paradox all the more profound. Mathematics is in fact only applicable if there is no definite causal connection between the events with which it deals. The obvious success of statistical mathematics seems however to suggest that there

*The Philosophy of Physical Science.

**Fundamental Theory.

is structure even in apparently fortuitous happenings. Although the outcome of each single event may be quite unpredictable in the ordinary way, an assembly of a large number of such events has structure which is amenable to mathematical treatment.

Mathematics, as we know it, does of course in the first instance arise as a free creation of the human mind. Geometry text books have been written without a single diagram and in recent years algebras and geometries have been created which are not derived from sense perception by abstraction. Some of these have already found application in science, others may still do so. Some in fact have been created to meet the needs of modern physics. And yet the examples quoted above may lead one to the conviction that the ideas so developed are in and behind the real world surrounding us. How could a science which apparently confines itself to the discovery of the mathematical structure of the universe set objective processes in train in nature if that structure were not an important and integral part of nature itself? The mathematical ideas reveal themselves in this way as part, perhaps the lowest part, of the intelligence and wisdom which lies behind the intelligence of the universe.

This conception is of course not a new one. Plato, who experienced ideas as fashioning, forming agencies in the world, rated mathematics so highly that he regarded its study as the necessary preliminary to an understanding of philosophy. Novalis, one of the most universal minds of the Romantic movement, said that when man was driven from paradise, God gave him number in its two forms of arithmetic and music as a reminder of his state before the fall. The discussion of the relation of mathematics to reality also figures in modern dissertations on philosophy and many modern mathematicians would hotly deny the claims here made for their subject.

If one recognises mathematical ideas and mathematical structure as existing independently of any human minds and as a part of the intelligence which orders the universe, the question, "Why do we teach mathematics?" is seen in a new light. Indeed the recognition can become the starting point for a revaluation of the tasks of education as a whole.

Two further questions present themselves. The first one is, How did these ideas come into existence, or, Who thinks these thoughts when human minds do not? The second one is, How is it that human minds can attain to mathematical ideas at all? The first question would lead us rather far from our immediate subject and will therefore not be considered further in this article. The second question on the other hand is most relevant and an answer must be attempted. The fact that one man can on the whole understand another's ideas is usually taken as evidence for the essential similiarity of the structures of different human minds. In fact our practice as certifying as abnormal anyone whose ideas cannot be readily comprehended by the majority of educated people is a practical application of this test. The same form of argument must lead us to the conviction that a mind which can grasp some of the ideas of the intelligence underlying the world must share some at least of the attributes of that intelligence. The exact relationship cannot of course be ascertained from the evidence and from the arguments presented above. In fact the mind of man at its present state of development can usually not penetrate further than to a general conviction of the existence of a universal mind and of its own kinship to it. Dr. Steiner has developed a teaching about the nature of man, which enables the student to penetrate further than this point. We are now going to quote from his work and would refer the reader to the editorial in the last number of CHILD AND MAN, where a justification for such a procedure has been set out.

Dr. Steiner outlines the nature of man somewhat as follows. In the main, man, as we know him at present, consists of two parts: a soul-spiritual part, which contains his eternal being, and a body which is supplied by the parents and is sustained by the natural world. The soul-spiritual part originates in the same region in which the powers which created and which maintain the universe are to be found and it is akin to those powers. Once this is accepted it is of course no longer surprising that the human mind is able to partake of some at least of the ideas behind nature.

During the embryonic development, the soul-spiritual part of man is linked to the body which it has to inhabit on the earth. It is however one of the great distinctions between man and animal that the former requires a much longer time after birth before he becomes a fully developed member of his species. The fact that full development is not reached until about twenty-one years after birth is recognised by many civilised peoples. This is the time which the soul-spirit part of man needs to take possession of the body fashioned for him and to find his way into human society on the earth. During a large part of this period the child and the young man or woman is guided by parents and educators and it should be the most solemn task of these adults to ensure that the process of incarnation proceeds in an orderly manner and that the result is a well balanced adult.

The process of incarnation may become abnormal in two ways. The soul-spiritual part may become too deeply immersed in the physical organism. It then tends to become subordinate to the needs and urges of that organism and is thus dominated by what should really be its instrument. Much that goes nowadays especially in more 'progressive' circles by the names of freedom and emancipation from outmoded convention is really nothing but slavery to the physical body. The second danger is no less real, although it often has less obvious results than the first. The incarnating soul-spirit may refuse to enter this world fully. In this case we get dreamers and ineffectual people.

Looking at education from the vantage point which we have now gained, we can no longer examine the various subjects only as to their usefulness in later life or even for their value in understanding the world. We must ask ourselves what their effect is likely to be on the process of incarnation.

Our discussion of mathematics shows that this subject rightly taught should have a very beneficial effect on the developing child. Its content derives from the origin of man's soul-spiritual nature; it can be found at work in the world around us and it orders and is applied to many of man's everyday activities. The subject should therefore have a harmonising influence in the process of incarnation, since it keeps the spirit of man in touch with his origin, and at the same time, by pointing to the interrelationship of physical objects and guiding their manipulation, draws his attention continually to the physical world. Thus through the study of mathematics the incarnating spirit of man is helped to avoid the two dangers mentioned above.

These considerations however also give a guide, at least in a general way, to how mathematics should be taught. If it is to harmonise, no aspect of the subject must be stressed at the expense of another. It would therefore be quite wrong to stress the practical application under the false impression that only in this way can the interest of less gifted pupils be kept alive. It has been the writer's experience that even very weak mathematicians can be interested in theoretical questions provided they are of real importance, and only of course if they are not expected to do the usual complicated problems found in text books. Examples of such theoretical questions will be given later. It is equally wrong to separate one's best pupils and to lead them by a tour de force into the higher reaches of the subject, leaving behind the more mundane application and not sparing the time to point out the appearance of arithmetic and geometrical law in nature, wherever the opportunity arises. There are essentially three aspects to our subject:—

1. The intrinsic theoretical content.
2. The appearance of mathematical law in nature.
3. The application of mathematics to commerce and technology.

All three must be given their due weight. A brilliant mathematician, who has no idea what laws underlie the formation of a musical scale and of harmony and who has never contemplated the geometrical forms found in shells, unicellular organisms, crystals and plants will have suffered in his education even if he turns out to be a successful research worker in his restricted field.

In a concluding article in the next number of CHILD AND MAN I shall deal in more detail with the curriculum in mathematics in a Rudolf Steiner school and how this is adapted to the needs of the developing child.

[*to be continued*].