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STAR POLYGONS

Revealing Cosmic Laws

by

Ernst Muller, Ph.D. (Vienna)

THE EVIDENT CONNECTION of certain geometrical figures with cosmic Laws of Nature is most clearly to be seen in the so-called" Star Polygons," which differ from the ordinary regular polygons (triangle, square, pentagon, etc.) by their repeated crossings, their folding back upon themselves. The simplest of star polygons is indeed avery ancient symbol of the Celtic and Pythagorean Mysteries. It is the so-called Pentagram (Fig. I), known also as the Pentalpha and by a variety of other names.

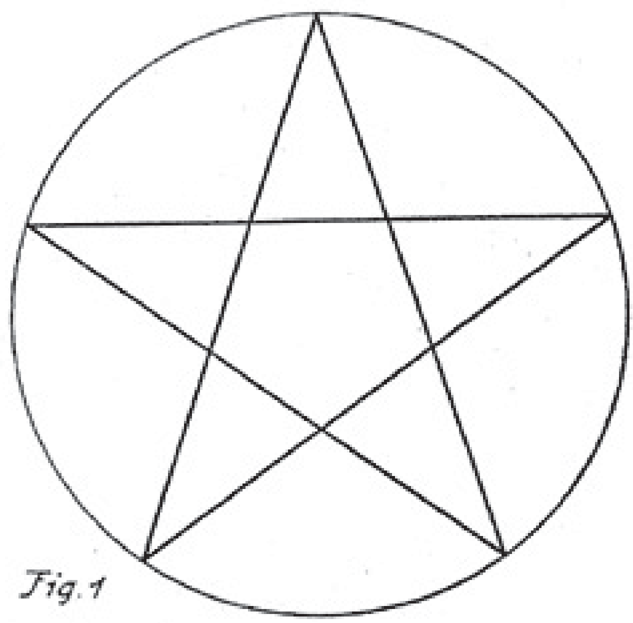

Fig. 1

A star polygon is formed if we mark out a number of points at equal distances along the circumference of a circle in such a way as to return to the starting-point, not after one but after several revolutions . We join the points, in the order taken, with straight lines or chords. We can then express every conceivable type of star polygon by means of a fraction, whereof the denominator which above all determines the "type" of the polygon-represents the number of times we have had to go round the circle, while the numerator gives the number of sides of the polygon. (The "simple" polygons are therefore of the type with denominator 1, the pentagram 2, 'and so on.)'.

We then obtain the polygon" *a/b* " by dividing the circumference of the circle into *a* parts and advancing *b* of these parts at a time. In general, *a*, and *b* are assumed to be relatively prime. If this is not so, the star polygon can be divided into a number of separate polygons interpenetrating one-another. The figures show the types 5/2 (Pentagram), 7/3, 7/2, 12/5, and 7/5. From their mode of origin, it is evident that the pairs of types 5/2 and 5/3, 7/3 and 7/4, or speaking generally *a/b* and *a/(a-b)*, represent the same star polygons. They differ only as regards the direction in which the star polygon has been described. Having once chosen a definite numeration corresponding to a certain way round the circle, the two types are related to one another as positive and negative. Thus the type 5/3, for example, may also be designated *minus* 5/2, and so on.

From a purely geometrical point of view, star polygons reveal other properties than ordinary or “simple” polygons it is worth noting that the star polygons of the most acute type, e.g. 7/3, 12/5, or, generally speaking, of the types *(2n+I)/n* and *2n/(n-I), to* which the pentagram and teven the simple triangle may be held to belong-invariably have the angular sum of 1800. Of the divisible star polygons the simplest, corresponding to the fraction 6/2, is the so-called Hexagram, known also as David’s Shield or as the Free-Masonic Symbol.

Peculiar three-dimensional forms are obtained if we consider the star polygons no longer as plane figures but if instead, at the points where. the sides cross, we let the one line alternately pass over or under the other**.** We obtain knots, formations knotted in multiple ways. The simplest knot is thus obtained from the Pentagram (Fig. 2). The Viennese mathematician *Oskar Simony*, who died in 1915, made an exhaustive study of these knotted forms. He discovered for example that certain properties of the knotted form *a/b*, laid out in a particular way, are in exact correspondence with the development of the fraction *a/q* as a “continued fraction.” (See Note 1.)


Fig. 2

It is most interesting that the orbits of the “ Moons” or “Satellites” are represented diagrammatically by means of lines which intertwine after the manner of the spatial knots derived from star polygons. In effect, after the corresponding planet has made a full revolution round the Sun, the satellite never returns exactly to its former place. The decisive points, where the effects of intertwining may be supposed to be concentrated, as it were,-namely the intersections of the Lunar orbit with the Ecliptic, have been named knots or *nodes* from time immemorial.

As to the sequence of the seven planets amongst themselves,. a certain star polygon is found to be especially important. In occult tradition, as is well-known, the following, with the Sun in the centre, are enumerated as the “seven planets”: Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon. The succession corresponds to the decreasing average distance of these” planets” from the Earth, only the names of Mercury and Venus having been interchanged in present-day nomenclature. If we compare with this order of the planets the succession of the corresponding days of the week, we obtain a star polygon of the type 7/3 (Fig. 3). This, the so-called “heptagram,” has been handed down to us from ancient Babylonian astronomy (see Note 2), to which indeed the division and nomenclature of our days of the week is ultimately due.

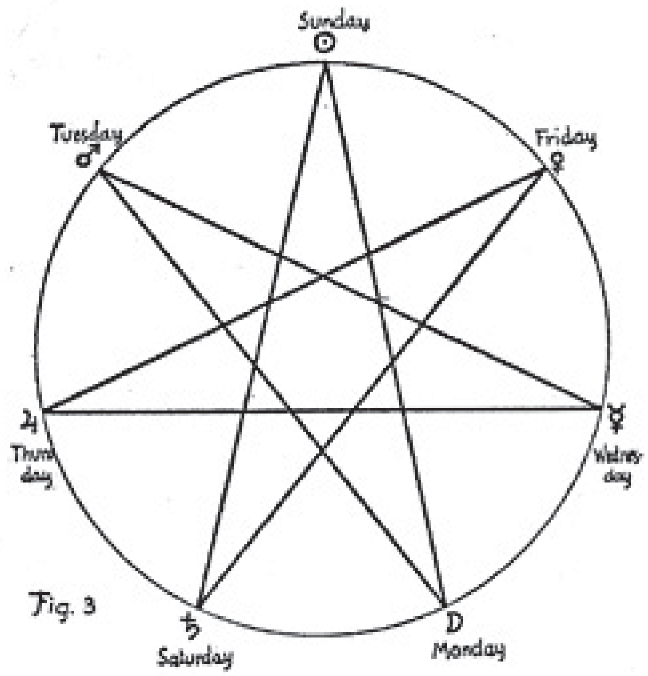

Fig. 3

The sequence of planets corresponding to the days of the week has also a purely astronomical significance, at least as regards the four planets Mars, Mercury, Jupiter and Venus (Mercury and Venus in their ancient meanings). It corresponds, in decreasing succession, to the periods of rotation after which the several planets return to the same positions in relation to the Earth *and* the Sun, these "helio-geocentric periods" being as follows:

♂ 778.7 days; ☿ 586.6 days; ♃ 398.7 days; ♀ 116 days.

The mutual ratios of these numbers even represent with fairly close approximation the simplest harmonic intervals of music, namely the octave (♂ to ♃); the fifth (☿ to ♃); the third of the octave (☿to ♀). The "velocities" of rotation represent, so to speak, the relative numbers of cosmic oscillations, if we regard it from a musical point of view.

The basic relationshIps within music itself also lend themselves to diagrammatic representation by means of star polygons. Fundamental to the harmonic relations is the well-known" circle of consecutive fifths," comprising all the notes of the chromatic scale (apart from the variety of octaves) in twefve successive 'fifths. On the other hand, for the melodic relations within the several scales, the natural "steps," namely the "tones and semitones," are decisive (e.g. C-D, or C-C# respectively).

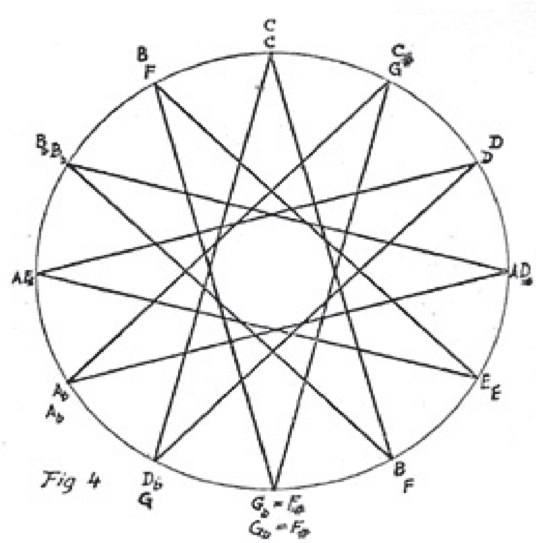

Fig 4

Taking our start from the circle of consecutive fifths (Fig. 4, inner designation of the 12 points, on the circle), we obtain the chromatic scale by following the lines of a star polygon of type 12/7. Conversely, if we begin with the chromatic scale (as indicated in the outer designation of the 12 points in Fig. 4), by following the same star polygon we obtain once more the succession of fifths.

It is more difficult to pass from the circle of consecutive fifths to the diatonic scale with its five tones and two semi-tones. Remembering however that two consecutive fifths make a tone (disregarding the octave once more), we obtain the diatonic scale by means of a star polygon of type 7/2 if we arrange seven successive fifths at equal intervals around a circle, for example FCGDAEB (Fig. 5). The ends, F and B in this example, the corresponding arc is indicated by a cross in the diagram-are then exactly half an octave (a so-called tritone or augmented fourth) apart. Following the lines of the star polygon from C onward, we obtain the scale of C-major. The central position of D in this diagram is due to the fact that the notes of the scale from D to its octave form a symmetrical sequence with respect to tones and semitones (occurring in the Dorian mode in the old Church music).

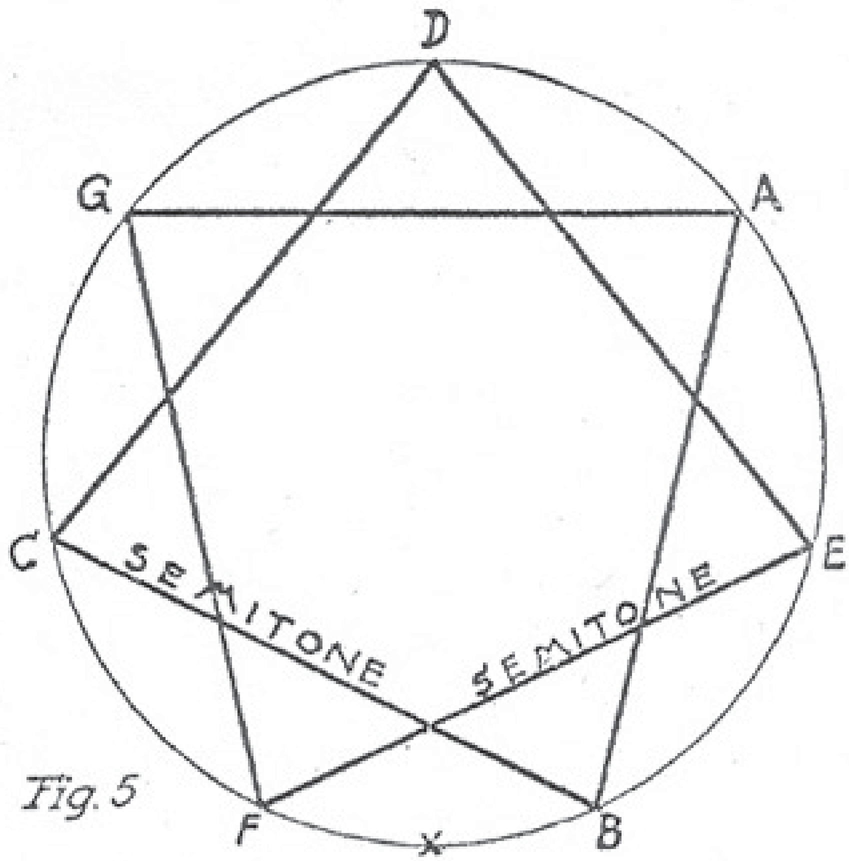

Fig. 5

In ancient pentatonic forms of music, among the Chinese for example, or in the Hebridean and in many German folksongs, we have an illustration of the simplest form of star polygon (Fig. 6).

The sequence of notes E-GAB-D is symmetrically related to the five successive fifths, GDAEB, shewn in the diagram. In view of its floating character, this sequence of notes may however also be taken in some other succession, e.g. DE GAB. (See Note 3.)

There is another interesting relation, between the star polygon 7/2 and the seven typical metals, with their traditional relation to the seven planets. Lead (Pb) is related to Saturn, tin (Sn) to Jupiter, iron (Fe) to Mars, gold (Au) to the Sun in the centre, copper (Cu) to Venus, mercury (Rg) to Mercury, and silver (Ag) to the Moon. There are certain polarities between the metals, as also between the planets, in their symmetrical relations to the Sun and to gold respectively. If we consider the .atomic weights of the seven metals we do not find any transparent relation between the numbers as svch. Nevertheless, the sequence of atomic weights arises out of the natural order of the metals-arranged, once more, according to their several planets-by means of a star polygon of the type 7/5, or 7/2, (Fig. 7).